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A New Assessment Method for the Bulk Modulus and the Poisson's Ratio of Porous Ceramics

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ABSTRACT: A new assessment method for the elastic moduli of porous materials is proposed. This method is based on a linear correlation between the Young's and the shear moduli of a given porous material. It is shown that such a correlation prevails for ceramics (Al_2O_3, ThO_2, and ZnO) and for glass (SiO_2) having Poisson's ratios of the solid bulk in the range of $0.15 < \nu_o \leq 0.36$. The outcome of the new assessment method is the use of either an elastic modulus or a sound velocity as the parameter for non-destructive assessment of the bulk and the Poisson's ratio of porous ceramics. These findings open a simplified route to quantitative non-destructive evaluation (QNDE) of the elastic moduli of porous materials.

KEYWORDS: quantitative non-destructive evaluation (QNDE), elastic modulus, elastic constants, sound wave velocity, porosity, ceramic, aluminum oxide, thorium oxide, zinc oxide, silicon dioxide, glass

The importance of the elastic moduli from the physical and the mechanical points of view was described by Wachtman [*1*]. For polycrystalline materials that are isotropic and homogeneous, only two independent elastic moduli are needed to describe the other elastic moduli [*1–7*]. In the past, attempts were made to correlate the shear modulus, *G*, and Young's modulus, *E*, of metals [*5*,*6*]. An approximate experimental value of $G/E \sim 3/8$ has been known for decades [*5*]. For F.C.C. and the B.C.C. metals, calculation of the interatomic potential using a two-body central force showed that *E* and *G* are proportional and that the ratios *G*/*E* are 0.396 and 0.367, respectively [*5*]. Gôrecki [*6*] looked for correlation between the elastic moduli of metals and found that *G*/*E* is 0.385 ± 0.052 for F.C.C., 0.357 ± 0.042 for B.C.C., and 0.389 ± 0.009 for H.C.P. metals [*6*]. For the case of ceramics, Poisson's ratio, ν, depends on the bond nature and is about 0.215 for covalent materials and 0.250 ± 0.059 for ionic metal oxides [*8*].

For the case of porous materials, attempts have been made, over the last five decades, to correlate the elastic moduli with density (or porosity) either experimentally or analytically. Experimental expressions for the elastic moduli-porosity correlation are numerous [*9–15*]. Analytical expressions exist for spherical [*7*,*8*,*16–24*] and non-spherical holes [*18–30*]. All these efforts were taken to validate the elastic moduli-density correlation so that the apparent density of a body would be the major parameter in the non-destructive evaluation of its elastic moduli. However, comparison of some of the models to the experimental results of ceramics leads to deviations, especially in the bulk modulus, *B*, and Poisson's ratio [*22*,*24*,*31*]. Moreover, the measured values of these elastic constants suffer from great scatter [*24*,*32*].

The purpose of the present paper is to propose a new assessment method for the elastic properties of porous materials, in particular the bulk modulus and Poisson's ratio of ceramics. We shall show, from an existing theoretical model, that over a large range of elastic moduli, a correlation between the shear and the Young's moduli of porous materials prevails. This finding is supported by existing experimental data. Thus, all the elastic properties of an isotropic, homogeneous, and porous material can be assessed from the measurement of a single elastic modulus or a sound velocity.

Determination of the Elastic Moduli

Assuming we know two independent elastic moduli, for example, *G* and *E*, the calculation of the other elastic constants is [*1*,*4*,*33–35*]:

$$B = EG/3(3G - E) \quad (1)$$

$$\nu = E/2G - 1 \quad (2)$$

The elastic properties can be determined by ultrasonic method [*3*,*4*,*33–35*]. For later use, the way of measuring the elastic moduli by this method is given here. The Young's and shear moduli are determined from the density, ρ, the shear wave velocity, V_S, and the longitudinal wave velocity, V_L [*4*,*33–35*]

$$G = \rho V_S^2 \quad (3)$$

$$E = \rho V_S^2\,(3V_L^2 - 4V_S^2)/(V_L^2 - V_S^2) \quad (4)$$

Studying Eq 3 and Eq 4 clearly shows that the first term on the right hand side of Eq 4 is the shear modulus. From Eq 2 it is easy to show that the second term in Eq 4 equals $2(1 + \nu)$.

Theoretical Background

The change of the effective bulk and shear moduli, B_e and G_e, respectively, of porous materials with pore concentration (porosity), *c*, and pore shape, was studied analytically by various researchers [*18*,*22–30*]. Comprehensive description of most of these models is given in Ref *24* and is beyond the scope of the present work. Some of these models [*8*,*23*,*28*,*31*] are based on the Mori-Tanaka effective stress field model [*36*]. Zaho et al. [*23*] and Dunn et al. [*8*] defended the validity of the Mori-Tanaka approach. Zaho et al. studied thoroughly the effect of pore concentration, shape, and orientation on the effective elastic moduli of porous material [*23*].

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Here the Zaho et al. approach is followed only for the case of 3D randomly oriented spheroid voids (pores). In general, the effective bulk and shear moduli of this class of porous material are [*8,23*]:

$$B_e = B_o/[1 + cp(\alpha,\nu_o)/(1 - c)] \quad (5)$$

$$G_e = G_o/[1 + cq(\alpha,\nu_o)/(1 - c)] \quad (6)$$

where B_o and G_o are the bulk and shear moduli of the solid bulk, p and q are functions of the pore aspect ratio, α, and of the Poisson's ratio of the solid bulk, ν_o. The effect of pore shape and concentration on the effective Poisson's ratio of porous solids, ν_e, was studied by Dunn et al. [*8*]. In that paper it was shown that ν_e depends more on ν_o and on α, and less on pore concentration. It was shown that a prolate pore affects ν_e in similar fashion as a spherical pore does, and that an oblate pore (disk or penny-shaped pore) strongly affects ν_e. Similar effect of pore shape on the elastic moduli is shown in other models [*22,28,30*]. For the sake of brevity in the present work the cases of oblate, needle shaped, and spherical pores only are treated.

For the case of oblate pores ($\alpha \ll 1$), q and p are [*23*]:

$$q = (8/15)\ 1/\pi\alpha\ (1 - \nu_o)(5 - \nu_o)/(2 - \nu_o); \quad p = (4/3)\ 1/\pi\alpha\ (1 - \nu_o^2)/(1 - 2\nu_o) \quad (7)$$

For the case of needle shaped pores ($\alpha \gg 1$), q and p are [*23*]:

$$q = (8/15)/(5 - 3\nu_o);\ p = (1/3)\ (5 - 4\nu_o)/(1 - 2\nu_o) \quad (8)$$

For the case of spherical pores, the expressions of B_e and G_e are slightly different [*23*]:

$$B_e = B_o\,[1 - c/(1 - c)a] \quad (9)$$

$$G_e = G_o\,[1 - c/(1 - c)b] \quad (10)$$

where a and b are [*23*]:

$$a = (1/3)(1 + \nu_o)/(1 - \nu_o);\ b = (2/15)(4 - 5\nu_o)/(1 - \nu_o) \quad (11)$$

From Eq 1 it is clear that the Young's modulus of porous material, E_e, can be retrieved when B_e and G_e are known and is

$$E_e = G_e[9B_e/(G_e + 3B_e)] \quad (12)$$

Note that the term in square brackets is $2(1 + \nu_e)$. According to Dunn et al. [*8*], for all materials having $-0.2 < \nu_o < 0.4$, ν_e changes by about ± 0.25 for the pore concentration range 0 to 1.0. Therefore for any material with ν_o in that range, the ratio, E/G, should change within margins of 0.25 (± 0.125) for the whole porosity range. To substantiate these analyses, see Fig. 1 to Fig. 3 below.

Figures 1*a* and 1*b* show the change of G_e/G_o and of E_e/E_o, respectively, versus porosity for materials with $\nu_o = 0.35$. Spherical pores, needle-shaped pores, and three cases of oblate pores are shown. From that figure it seems that G_e and E_e do have a similar dependence on c. Figure 2 shows the change of G_e/G_o versus E_e/E_o for all these cases. Figure 3 shows the change of G_e/G_o versus E_e/E_o as a function of ν_o for spherical pores, Fig. 3*a*, and for oblate pores having $\alpha = 0.10$, Fig. 3*b*. From Fig. 2 and Fig. 3 it seems that regardless of pore shape the relation of G_e/G_o versus E_e/E_o can be fairly approximated by a linear expression for ceramics having a Poisson's ratio of $0.15 < \nu_o < 0.35$.

The Proposed Assessment Method

The proposed assessment method for the effective elastic constants of porous ceramics having a Poisson's ratio of $0.15 < \nu_o < 0.35$ consists of two steps:

1. A linear correlation between the measured shear modulus, G_e (or G_e/G_o) and the Young's modulus, E_e (or E_e/E_o), of porous material is put forward:

$$G_e = a_G E_e + b_G;\ G_e/G_o = a_G^*\,E_e/E_o + b_G^* \quad (13)$$

where a_G and b_G (or a_G^* and b_G^*) are constants. The interrelationships between these coefficients and their relation to the elastic

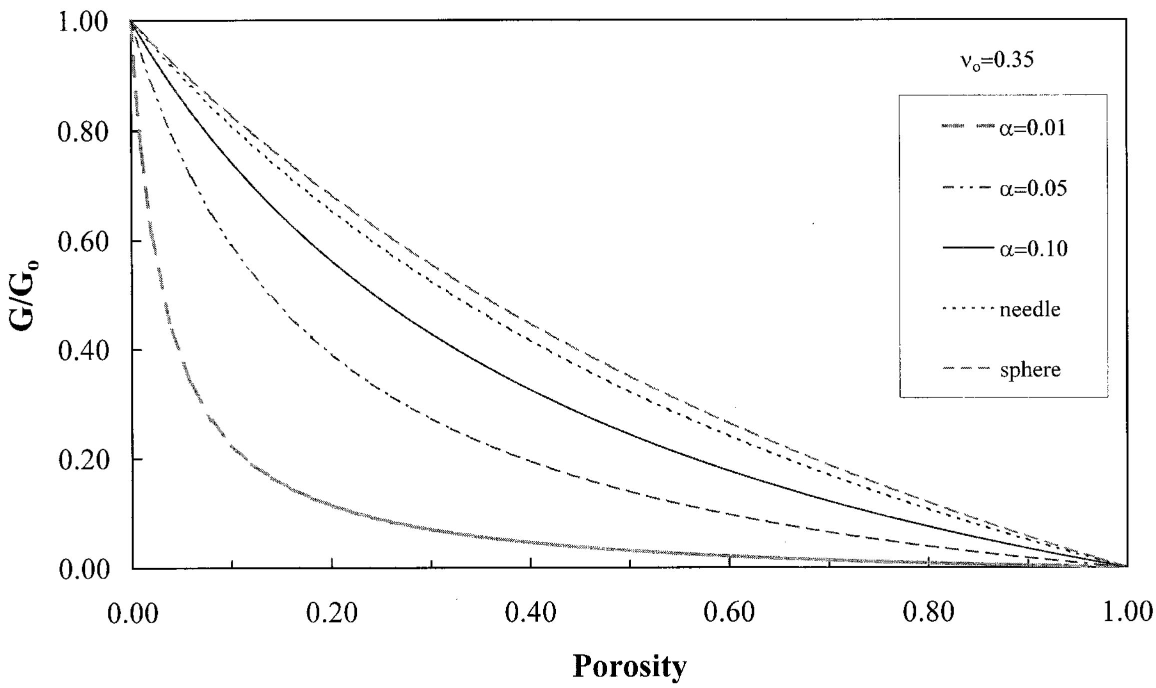

FIG. 1*a*—*The change of the normalized shear modulus versus porosity for materials having a Poisson's ratio of $\nu_o = 0.35$. The equations for the various pores based on Zaho et al. [23].*

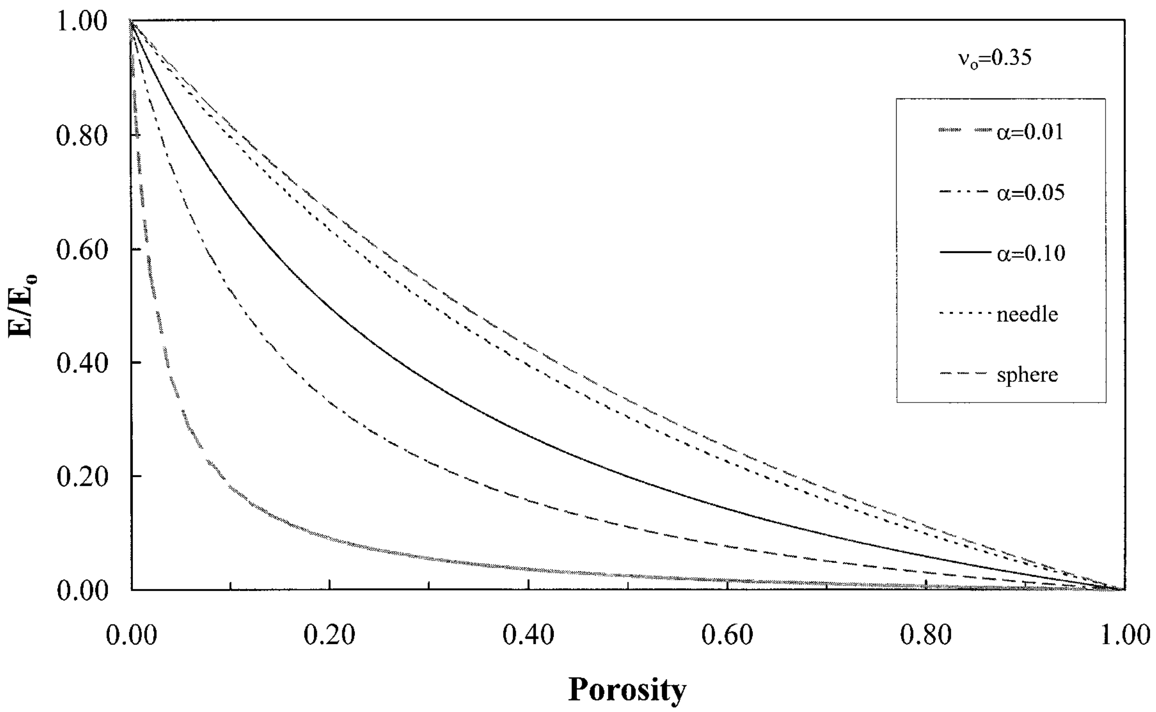

FIG. 1*b—The change of the normalized Young's modulus versus porosity for materials having a Poisson's ratio of* ν_o *= 0.35. The equations for the various pores based on Zaho et al. [23].*

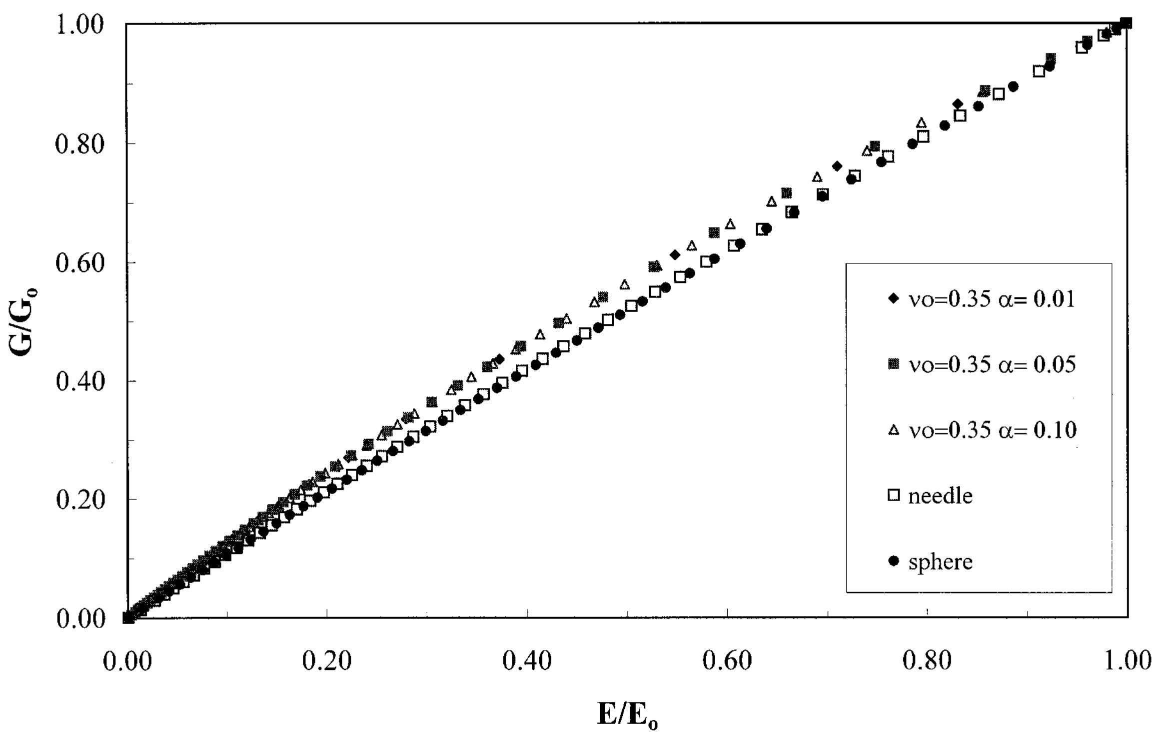

FIG. 2—*The change of the normalized shear modulus versus the normalized Young's modulus for the cases of oblate, needle-shaped and spherical pores shown in Fig. 1.*

moduli of the solid bulk are given in the Appendix. The thus-assessed shear modulus is referred to hereafter as G_{ass}.

2. Assess the bulk modulus or the Poisson's ratio from G_{ass}. This causes an averaging effect yielding less scatter of these elastic properties.

Moreover, provided that the above correlation exists, it is easy to show, from Eq 3 and Eq 4, the interdependence of the longitudinal and the shear sound wave velocities. Therefore, since the elastic moduli depend on pore shape, it is logical to plot the bulk modulus or Poisson's ratio against either E_e or G_e or more simply either V_L

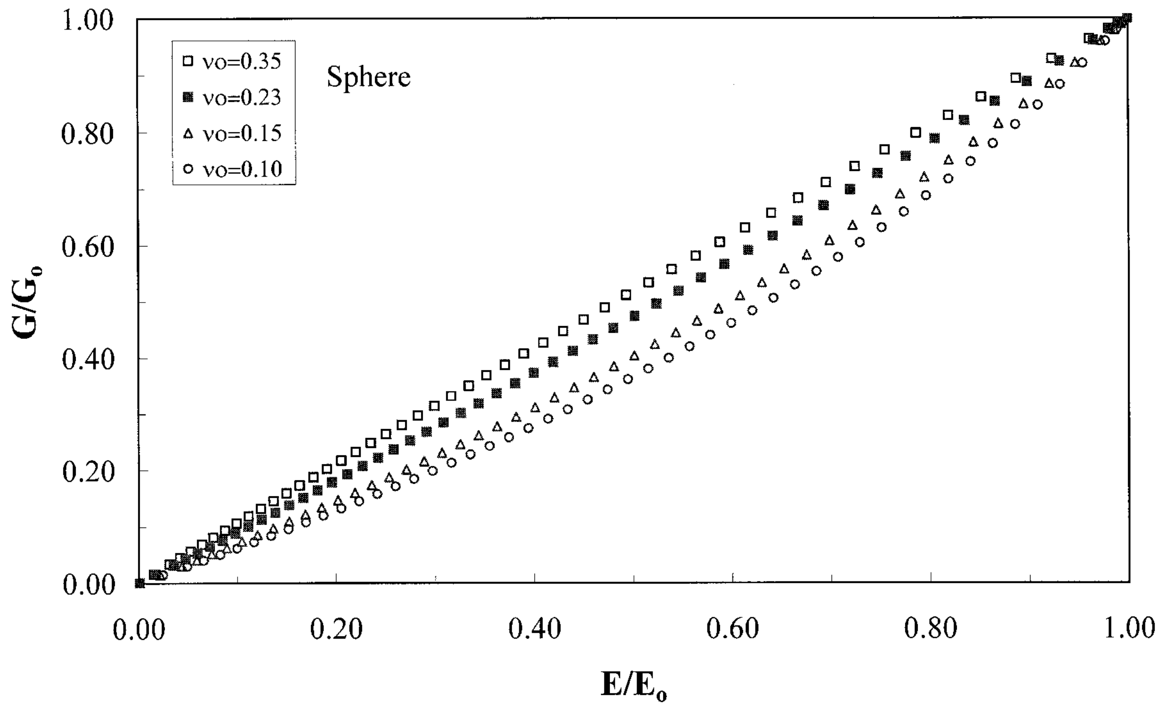

FIG. 3*a*—*The change of the normalized shear modulus versus the normalized Young's modulus for spherical pores in materials with various ν_o. The equations based on Zaho et al. [23].*

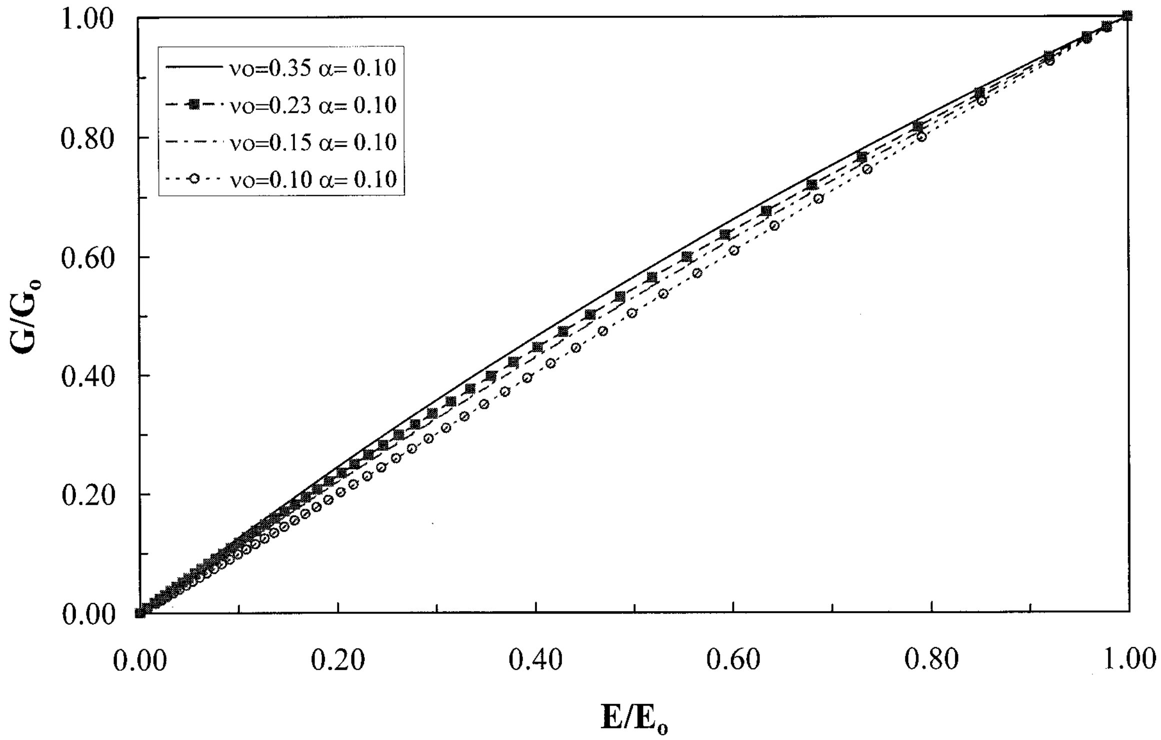

FIG. 3*b*—*The change of the normalized shear modulus versus the normalized Young's modulus for oblate pores with $\alpha = 0.1$ in materials with various ν_o. The equations are from Zaho et al. [23].*

or V_S. This approach provides a simplified way of quantitative nondestructive evaluation (QNDE) of the elastic moduli of porous ceramics.

Results and Discussion

The Shear Modulus—Young's Modulus Correlation in Real Ceramics

Figure 4 shows a plot of G_e/G_o versus E_e/E_o for Si_2O, Al_2O_3, ThO_2, and ZnO, materials having ν_o in the range of 0.17 to 0.36. The authors realize that silica is a glass and is given here as a complementary case. As to the data points, they were taken from the literature respectively with the material [*37–39,31*]. (Note that in the paper of Adachi et al. [*37*] there is a typographical error, the bulk and shear moduli were interchanged). The data of Spinner et al. on ThO_2 was retrieved from Dean [*22*] and consisted of two types: Type I (initial powder 0 to 2 μm) and Type II (initial particle size 2 to 4 μm). Figure 4 shows that a linear relationship prevails between G_e/G_o and E_e/E_o for the range $0 < E_e/E_o \leq 1.0$. Figure 4 resembles Fig. 3*b* more than Fig. 3*a*. One should note, however, that Fig. 3*a* shows more linear over concave behavior as ν_o increases from 0.1 to 0.35. Table 1 gives the coefficients a_G^* and b_G^* of Eq 13 and the coefficient of correlation R^2 for these four cases. From Table 1, one can see that the R^2 values are very high, typically above 0.998, and that a_G^* and b_G^* obey:

$$a_G^* + b_G^* \cong 1.0 \tag{14}$$

as should be from the calculation in the Appendix. Therefore, there is a consistency between calculation and experimental results.

The Poisson's Ratio Against Porosity, Elastic Modulus or Longitudinal Sound Velocity

An advantage of introducing of G_{ass} from Eq 13 into Eq 1 and Eq 2 is the possibility to assess the Poisson's ratio and the bulk modulus, respectively.

In the example below, the V_L and V_S values were calculated from the measured data. For ThO_2 the theoretical density is

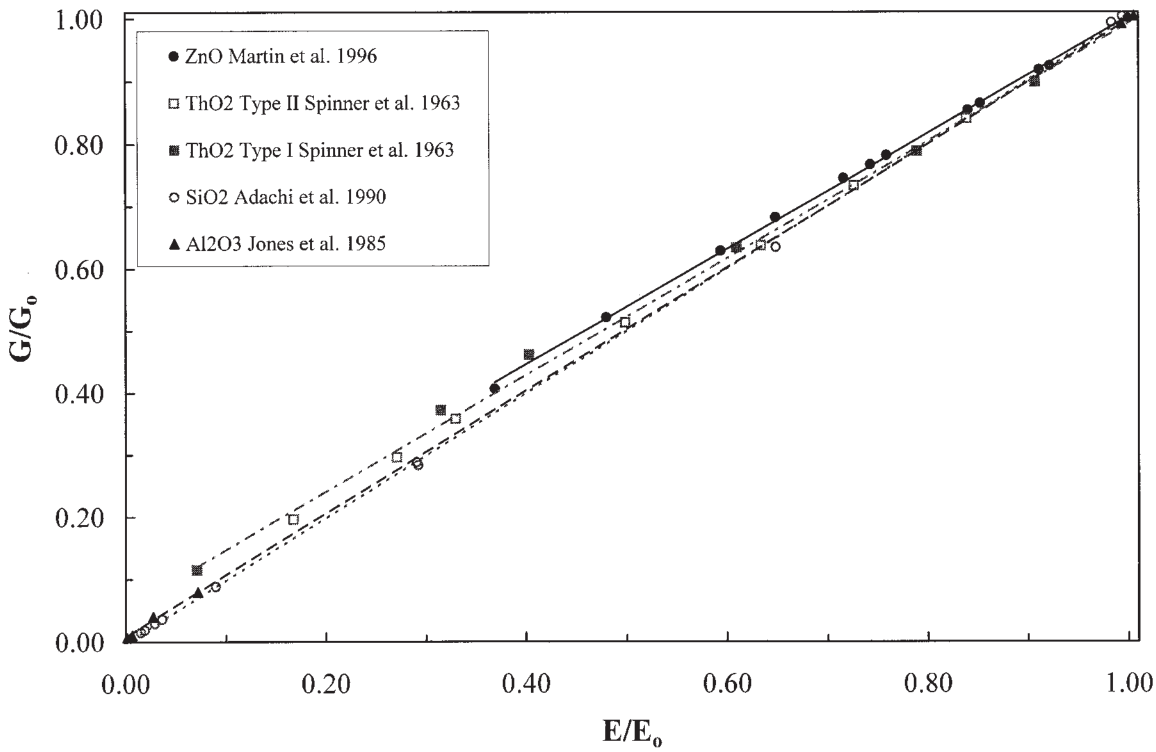

FIG. 4—*The change of the normalized shear modulus versus the normalized Young's modulus in real ceramics Al_2O_3, ThO_2, and ZnO and glass SiO_2. (Data of Refs* 38, 39, 31, *and* 37, *respectively, with the material.)*

TABLE 1—*The coefficients of the linear relation between* G_e/G_o *and* E_e/E_o *(Eq 13) and the coefficient of correlation* R^2 *for the four materials in Fig. 4. Also given are, for the elastic constants of the solid bulk,* ν_o *and* E_o *that were used and their sources. The values used are: for ceramics, the Voigt Reuess Hill values, and for the glass, the average for fused silica from Spinner* [40].

Material	a_G	b_G	R^2	ν_o	E_o, GPa	Reference
SiO_2	1.0054	−0.0029	0.9998	0.168	72.9	40
Al_2O_3	0.9905	0.0077	1.000	0.232	402.7	2
ThO_2	0.9403	0.0525	0.9978	0.285	249.1	39
ZnO	0.9281	0.0747	0.9992	0.356	123.6	2

10 000 kg m^{-3} [*41*]. The measured Poisson's ratio, ν_e, was calculated from the measured E_e and G_e and the assessed Poisson's ratio as described above. Figure 5*a* shows the measured Poisson's ratio against the porosity for ThO_2. Figure 5*b* shows the measured values (filled symbols) and the assessed values (open symbols) against E_e/E_o and Fig. 5*c* the same data points against V_L/V_{Lo}. Three features, i-iii, are observed in Fig. 5:

(i) Monotonic change prevails between Poisson's ratio (measured and assessed) and E_e/E_o or V_L/V_{Lo} as opposed to their change with porosity.
(ii) The Poisson's ratio of ThO_2 can be non-destructively evaluated in a reliable manner in the range of $0.1 \leq E_e/E_o \leq 1.0$ and of $0.3 \leq V_L/V_{Lo} \leq 1.0$.
(iii) Negative Poisson's ratio for ThO_2 samples is assessed by the proposed method.

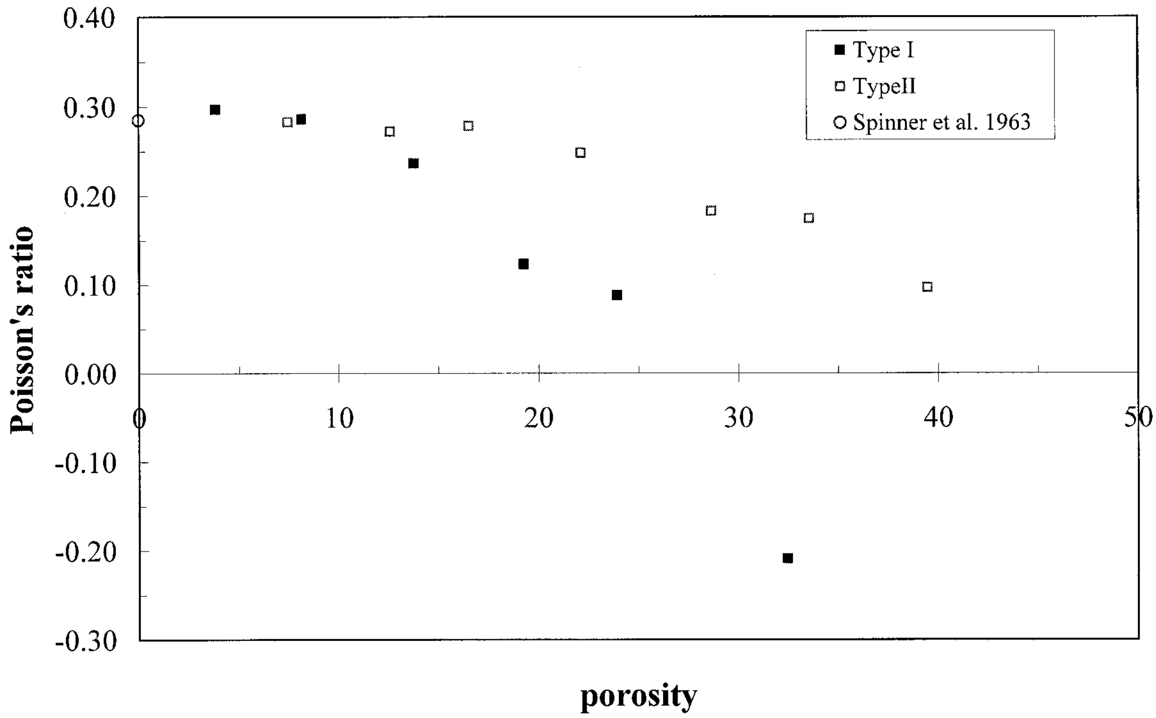

FIG. 5*a—The change of the Poisson's ratio with porosity of the two types of ThO_2 powders (measured data of Spinner et al.* [39]*.*

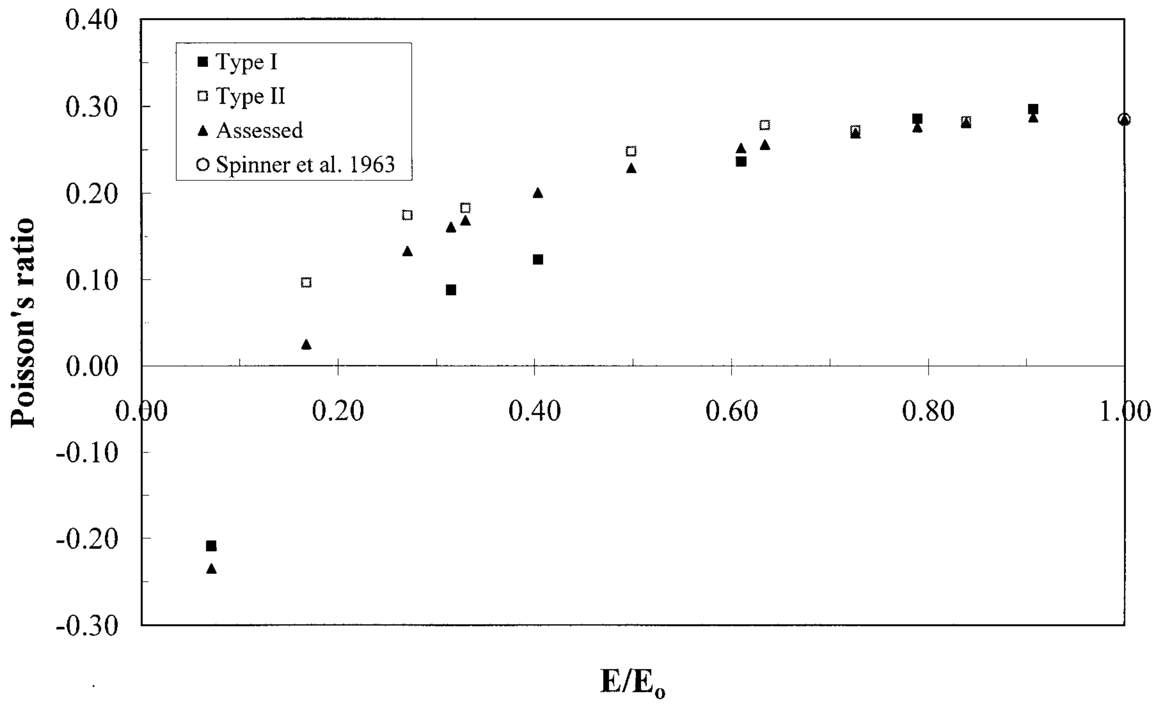

FIG. 5*b—The change of the Poisson's ratio with the normalized Young's modulus. Measured values (open and filled squares and open circle) and assessed values (filled triangles). Same data points as in Fig. 5*a.

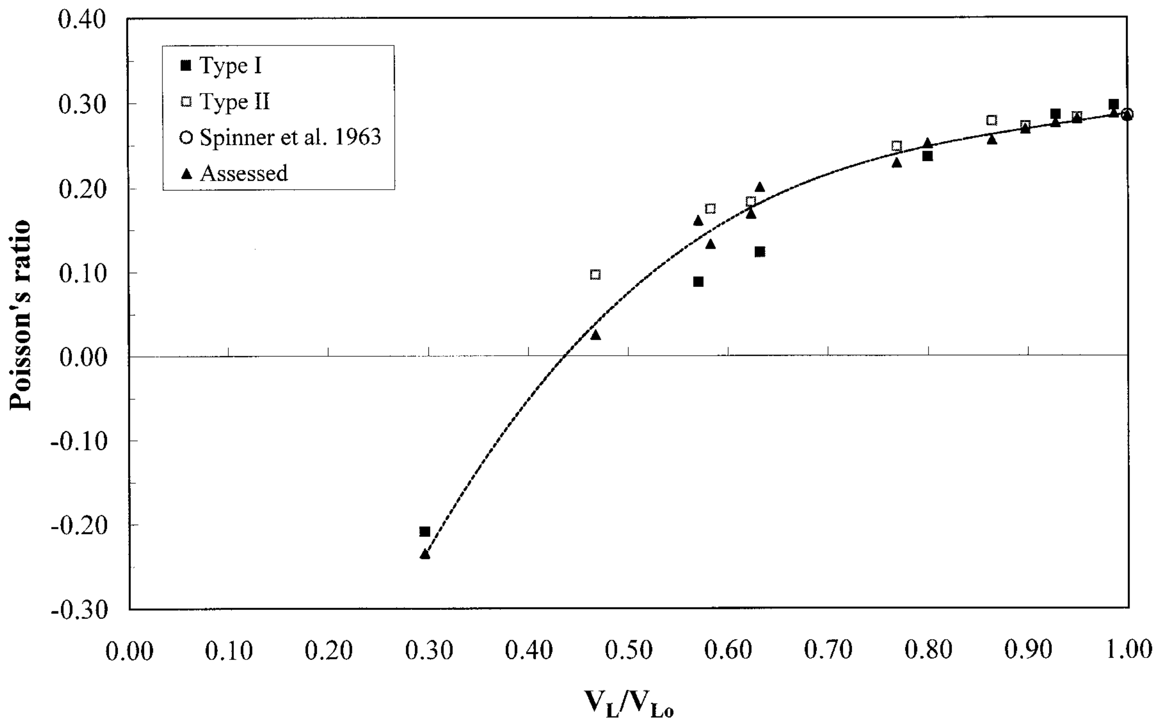

FIG. 5*c*—*The change of the Poisson's ratio with the normalized longitudinal sound velocity. Measured values (filled symbols) and assessed values (open symbols). Same data points as in Fig. 5*a.

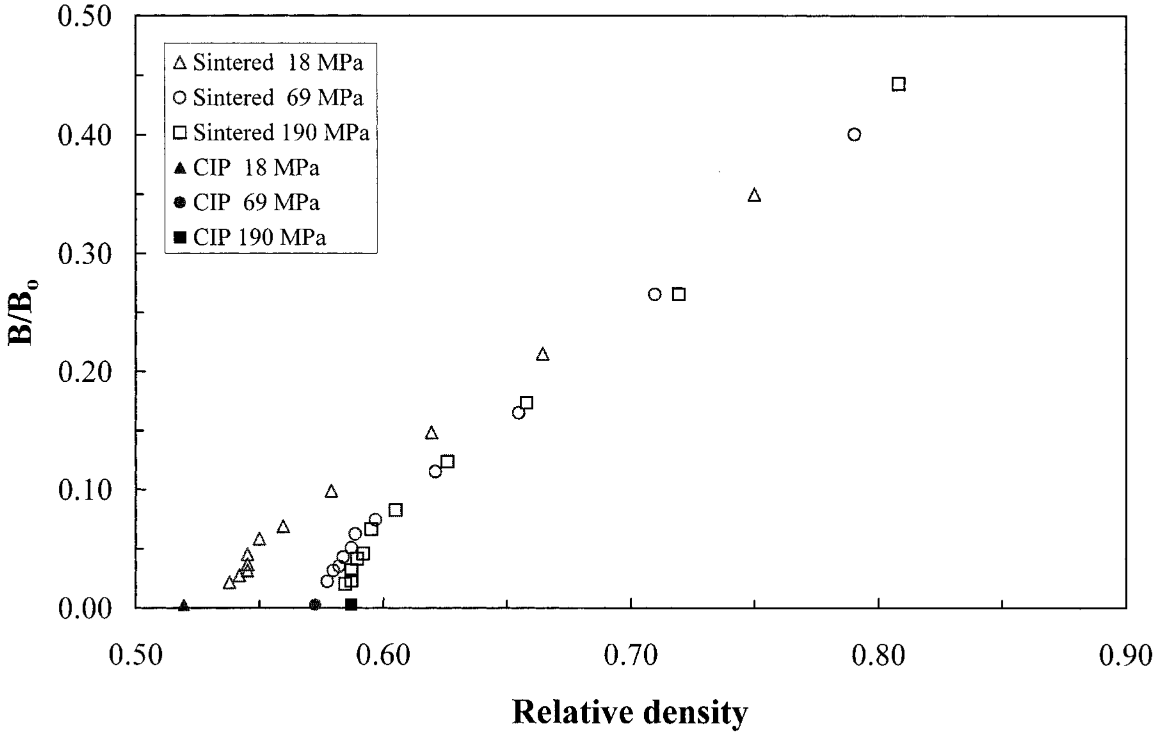

FIG. 6*a*—*The change of the measured normalized bulk modulus of three groups of Al_2O_3 samples versus the relative density (data of Green et al. [42]).*

The Measured and Assessed Bulk Modulus Against Porosity or Longitudinal Sound Velocity

In the example below, the V_L and V_S values were calculated from the measured elastic moduli data of Green et al. [*42*]. For Al_2O_3 the theoretical density is 3986 kg m^{-3} [*2*]. Figure 6*a* shows the change of the normalized bulk modulus of Al_2O_3 against the relative density of samples that were cold compacted (at 18, 69, and 190 MPa) and sintered [*42*]. Figure 6*b* shows the change of the same property with the normalized longitudinal sound wave velocity. Note that all the data points are aligned along a monotonic line resembling a parabolic curve. In general the correlation between the normalized

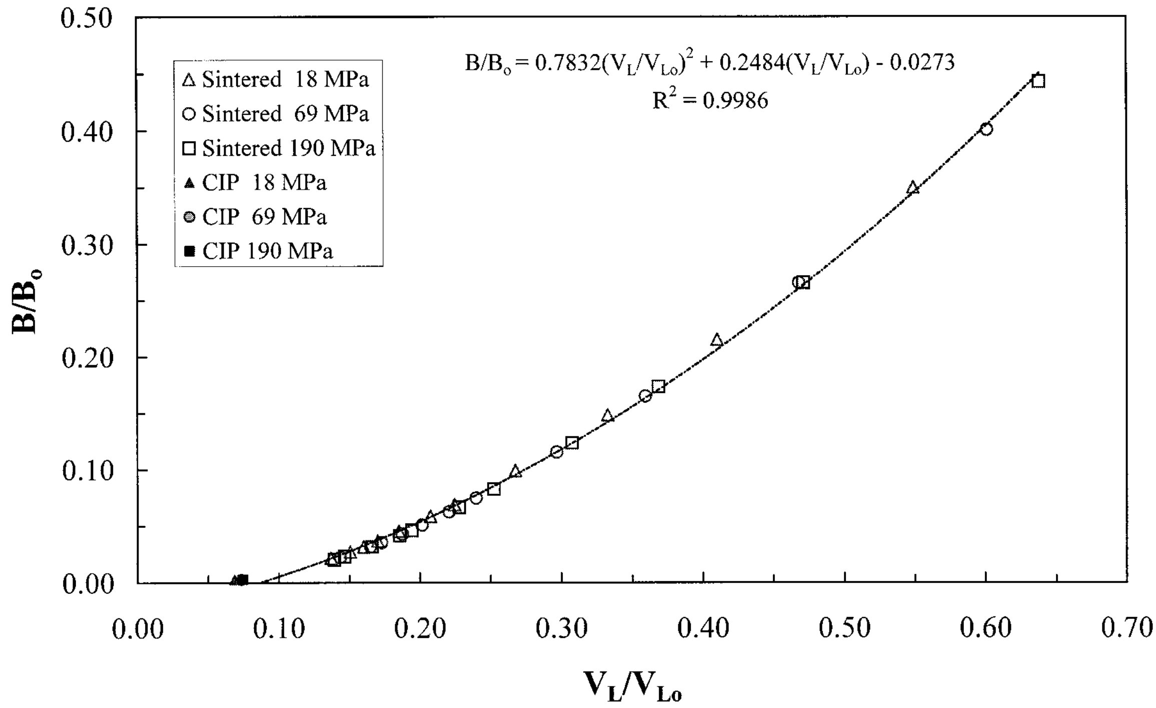

FIG. 6*b—The change of the measured normalized bulk modulus of three groups of Al_2O_3 samples versus the normalized longitudinal sound velocity. Same data points as in Fig. 6*a.

elastic moduli (E_e/E_o, G_e/G_o, and B_e/B_o) and the normalized longitudinal sound wave velocity can be expressed as polynomial, e.g., as a second power of V_L/V_{Lo}:

$$M/M_o = a_{VM}(V_L/V_{Lo})^2 + b_{VM}(V_L/V_{Lo}) + c_M \quad (15)$$

where a_{VM}, b_{VM}, and c_M are constants for each modulus, which can be derived either from a_G^* and b_G^* or simply from a best-fit analysis.

Similar results on the other elastic properties than B_e/B_o, although not reported here, exist in porous Al_2O_3, SiO_2, ZnO, MgO, $MgAl_2O_4$, Y_2O_3, and other ceramics. The proposed assessment method is the sole method that the authors are aware of that enables a relatively accurate assessment of the Poisson's ratio of porous ceramics including negative values as in the data of Spinner et al. [*39*] and Jones et al. [*38*].

Conclusions

A linear correlation is put forward between the Young's and the shear moduli of porous materials. This is backed by an existing analytical model and verified by experimental data points on ceramics and glass having Poisson's ratios of the solid bulk in the range of $0.15 < \nu_o \leq 0.36$. Using an assessed elastic modulus enables one to assess the bulk modulus and the Poisson's ratio. Quantitative non-destructive evaluation (QNDE) of the elastic properties is suggested with Young's modulus as a parameter for evaluation. Moreover, the outcome of the linear correlation between *G* and *E* yields the interrelation between the shear and the longitudinal sound wave velocities. It is shown that sound velocity also can be used as the parameter for assessment of the bulk and the Poisson's ratios of porous ceramics. These findings are an important simplification of the QNDE of the elastic moduli of porous ceramics.

Acknowledgments

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APPENDIX

The Poisson's ratio of the bulk solid, ν_o, is:

$$\nu_o = E_o/2G_o - 1 \quad (A1)$$

The Poisson's ratio of porous materials can be assessed, ν_{ass}, providing G_{ass} is known:

$$G_{ass} = a_G E_e + b_G \quad (A2)$$

By assuming that ν_e should be equal to ν_{ass} we get:

$$\nu_e \cong \nu_{ass} = E_e/2(a_G E_e + b_G) - 1 \quad (A3)$$

Since this should be also true for the solid bulk we get:

$$\nu_o = E_o/2(a_G E_o + b_G) - 1 \quad (A4)$$

Therefore the relation between a_G and b_G is:

$$a_G = 1/[2(1 + \nu_o)] - b_G/E_o \quad (A5)$$

From (A3) b_G depends also on the elastic properties of the porous material:

$$b_G = E_e[1 - 2(1 + \nu_e)a_G]/2(1 + \nu_e) \quad (A6)$$

Introducing (A3) into (A6) gives:

$$b_G = E_e(\nu_o - \nu_e)/[2(1 + \nu_o)(1 + \nu_e)(1 - E_e/E_o)] \quad (A7)$$

In order to solve (A5) and (A7) it is clear that one needs two elastic moduli of the solid bulk, e.g., ν_o and E_o and *at least one measurement of two independent moduli* of a porous sample, e.g., E_e and ν_e.

For the case of the relation between normalized elastic moduli:

$$G_e/G_o = a_G^* E_e/E_o + b_G^* \quad \text{(A8)}$$

Since we have to keep both sides of the equation equal from Eq A1:

$$G_e/G_o = a_G 2(1 + \nu_o)E_e/E_o + 2(1 + \nu_o)b_G/E_o \quad \text{(A9)}$$

hence

$$a_G^* = 2a_G(1 + \nu_o); \qquad b_G^* = 2b_G(1 + \nu_o)/E_o \quad \text{(A10)}$$

Combining Eq A5 with Eq A10 yields:

$$a_G^* + b_G^* = 1.0 \quad \text{(A11)}$$

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